LIBERTY PAPER SET

STD. 10 : Mathematics (Basic) [N-018(E)]

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 8**

Section-A

1. (B) $\frac{4}{3}$ **2.** (B) 8 **3.** (D) 120 **4.** (C) −1 **5.** (B) $\frac{3}{4}$ **6.** (C) $\frac{x_i - a}{h}$ **7.** 2 **8.** Parabola **9.** $\frac{1}{3}$ **10.** −5 **11.** 40° **12.** 2 **13.** False **14.** False **15.** True **16.** True **17.** −5 **18.** 120° **19.** $\frac{1}{7}$ **20.** 30 – 40 **21.** (a) $\frac{4}{3}\pi r^3$ **22.** (b) $2\pi r^2$ **23.** (c) πr^2 **24.** (a) $\frac{\pi r\theta}{180}$

Section-B

25. Let the quadratic polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$\therefore\ \alpha + \beta = \frac{-5}{2} = \frac{-b}{a}$ and $\alpha\beta = \frac{3}{2} = \frac{c}{a}$

$\therefore\ a = 2,\ b = 5$ and $c = 3$

So, one quadratic polynomial which fits the given conditions is $2x^2 + 5x + 3$. You can check that any other quadratic polynomial that fits these conditions will be of the form $k(2x^2 + 5x + 3)$, where k is real.

26. Here $a = 2,\ b = 3,\ c = -5$ and d = 8

Sum of the zeroes $= -\frac{b}{a} = -\frac{3}{2}$ and

Product of the zeroes $= -\frac{d}{a} = -\frac{8}{2} = -4$

27. $\sqrt{3}x^2 - 5x + 2\sqrt{3} = 0$

$\therefore\ \sqrt{3}x^2 - 3x - 2x + 2\sqrt{3} = 0$

$\therefore\ \sqrt{3}x(x - \sqrt{3}) - 2(x - \sqrt{3}) = 0$

$\therefore\ (x - \sqrt{3})(\sqrt{3}x - 2) = 0$

$\therefore\ x - \sqrt{3} = 0$ OR $\sqrt{3}x - 2 = 0$

$\therefore\ x - \sqrt{3}$ OR $x - \frac{2}{\sqrt{3}}$

28. Here, AP : 10, 7, 4, ..., –62

$a = 10,\ d = 7 - 10 = -3,\ a_n = -62$

Now, $a_n = a + (n - 1)d$

$\therefore\ -62 = 10 + (n - 1)(-3)$

$\therefore\ -62 - 10 = (n - 1)(-3)$

$\therefore\ \frac{-72}{-3} = n - 1$

$\therefore\ n - 1 = 24$

$\therefore\ n = 25$

So, there are 25 terms in the given AP.

Now 11th term from last term (25th term) is 15th term.

$\therefore\ a_{15} = a + 14d$

$\therefore\ a_{15} = 10 + 14(-3)$

$\therefore\ a_{15} = 10 - 42$

$\therefore\ a_{15} = -32$

29. The AP formed by the factor of 4 between 10 and 250 is 12, 16, 20, ..., 248.

$\therefore\ a = 12,\ d = 16 - 12 = 4,\ a_n = 248$

$a_n = a + (n - 1)d$

$\therefore\ 248 = 12 + (n - 1)4$

$\therefore\ 248 - 12 = (n - 1)4$

$\therefore\ \frac{236}{4} = n - 1$

$\therefore\ n - 1 = 59$

$\therefore\ n = 60$

Thus, there are 60 multiples of 4 lie between 10 and 250.

30. $\left(\frac{m}{2}, 5\right) = \left(\frac{-6-2}{2}, \frac{7+3}{2}\right)$

$\therefore\ \frac{m}{2} = \frac{-6-2}{2}$

$\therefore\ m = -8$

31. Suppose, the ratio in which line segment joining A (–3, 10) and B (6, – 8) is divided by point P (–1, 6) is $m_1 : m_2$.

Co-ordinates of point P = $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$\therefore\ (-1, 6) = \left(\frac{m_1(6) + m_2(-3)}{m_1 + m_2}, \frac{m_1(-8) + m_2(10)}{m_1 + m_2}\right)$

$\therefore\ (-1, 6) = \left(\frac{6m_1 - 3m_2}{m_1 + m_2}, \frac{-8m_1 + 10m_2}{m_1 + m_2}\right)$

$\therefore\ -1 = \frac{6m_1 - 3m_2}{m_1 + m_2}$

$\therefore\ -m_1 - m_2 = 6m_1 - 3m_2$

$\therefore\ -m_1 - 6m_1 = -3m_2 + m_2$

$\therefore\ -7m_1 = -2m_2$

$\therefore\ \frac{m_1}{m_2} = \frac{2}{7}$

Hence, the point P will divide AB into a 2 : 7 ratio.

32. $cos^2 A = 1 - sin^2 A\ (\because\ cos^2A + sin^2 A = 1)$

$\therefore\ cos\ A = \sqrt{1 - sin^2 A}$

$\therefore\ tan\ A = \frac{sin\ A}{cos\ A} = \frac{sin\ A}{\sqrt{1 - sin^2 A}}$

$\therefore\ sec\ A = \frac{1}{cos\ A} = \frac{1}{\sqrt{1 - sin^2 A}}$

33. $\cos 60° \cdot \sin 30° + \sin 60° \cdot \cos 30°$

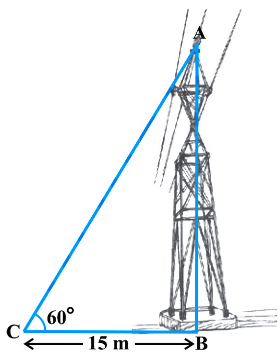
$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= \frac{1+3}{4}$$

$$= \frac{4}{4} = 1$$

34.



Here, AB represents the tower, CB = 15 is the point from the tower and $\angle ACB$ is the angle of elevation = 60°.

Now, $\tan 60° = \dfrac{AB}{BC}$

$$\therefore \sqrt{3} = \frac{AB}{15}$$

$$\therefore AB = 15\sqrt{3} \text{ m}$$

Hence, the height of the tower is $15\sqrt{3}$ m.

35. Cylinder   Cone

r = 12 cm r = 12 cm

h = 11 cm l = 13 cm

Total area of the box

= CSA of cylinder + CSA of cone

= $2\pi rh + \pi rl$

= $\pi r(2h + l)$

= $\frac{22}{7} \times 12 \times [2 (II) + 13]$

= $\frac{22}{7} \times 12 \times 35$

= 1320 cm^2

36. Here, d = 5.6 m, $r = \frac{d}{2} = \frac{5.6}{2} = 2.8$ m

h = 25 cm = 0.25 m

Volume of sand required

= Volume of cylinder so formed

$= \pi r^2 h$

$= \frac{22}{7} \times 2.8^2 \times 0.25$

$= \frac{22}{7} \times \frac{28}{10} \times \frac{28}{10} \times \frac{25}{100}$

$= \frac{22 \times 7 \times 4 \times 28 \times 25}{7 \times 10 \times 10 \times 25 \times 4}$

$= \frac{22 \times 28}{100}$

$= \frac{616}{100}$

$\therefore$ Required sand = 6.16 m^3

37. $\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$\therefore 211 = 225 + \frac{-7}{25} \times h$

$\therefore 211 - 225 = \frac{-7}{25} \times h$

$\therefore -14 \times \frac{25}{-7} = h$

$\therefore h = 50$

38. $2x + y = 7$ .....(1)

$x - 2y = 6$ .....(2)

$\therefore x = 6 + 2y$ .....(3)

Put eq^n (3) in eq^n (1)

$2x + y = 7$

$\therefore 2(6 + 2y) + y = 7$

$\therefore 12 + 4y + y = 7$

$\therefore 5y = 7 - 12$

$\therefore 5y = -5$

$\therefore y = -1$

Put $y = -1$ in eq^n (3)

$x = 6 + 2y$

$\therefore x = 6 + 2(-1)$

$\therefore x = 6 - 2$

$\therefore x = 4$

Therefore, the solution is $x = 4$ and $y = -1$

39. Suppose, the numerator is x and the denomenator is y.

the fraction = $\frac{x}{y}$

According to the first condition;

$$\frac{x+1}{y-1} = 1$$

$\therefore\ x + 1 = y - 1$

$\therefore\ x - y = -2$ ...(1)

According to the second condition;

$$\frac{x}{y+1} = \frac{1}{2}$$

$\therefore\ 2x = y + 1$

$\therefore\ 2x - y = 1$ ...(2)

Subtracting equation (1) and (2),

$$\begin{array}{rcl} x - y &=& -2 \\ 2x - y &=& 1 \\ - \quad + & & - \\ \hline \end{array}$$

$\therefore\ -x = -3$

$\therefore\ x = 3$

Put $x = 3$ in equation (1)

$x - y = -2$

$\therefore\ 3 - y = -2$

$\therefore\ y = 5$

Hence, the fraction = $\frac{x}{y} = \frac{3}{5}$

40. $a_3 = 15$, $S_{10} = 125$, $d =$ ____, $a_{10} =$ ____

$a_3 = a + 2d = 15$ ...(1)

$\therefore\ S_n = \frac{n}{2}\,[2a + (n - 1)\,d]$

$S_{10} = \frac{10}{2}\,[2a + (10 - 1)\,d]$

$\therefore\ 125 = 5\,(2a + 9d)$

$\therefore\ \frac{125}{5} = 2a + 9d$

$\therefore\ 2a + 9d = 25$ ...(2)

Equation (1) multiply by 2 & subtracting equation (2) from (1)

$$\begin{array}{rcl} 2a + 4d &=& 30 \\ 2a + 9d &=& 25 \\ - \quad - & & - \\ \hline \end{array}$$

$\therefore\ -5d = 5$

$\therefore\ d = -1$

Put $d = -1$ in equation (1), we get,

$a + 2d = 15$

$\therefore\ a + 2(-1) = 15$

$\therefore\ a - 2 = 15$

$\therefore\ a = 17$

Now, $a_n = a + (n - 1)\,d$

$\therefore\ a_{10} = 17 + (10 - 1)\,(-1)$

$\therefore\ a_{10} = 17 - 9$

$\therefore\ a_{10} = 8$

41. Suppose, the line dividing the line sugement AB connection A (–1, 7) and B (4, 2) in the ratio $m, : m2 = 3 : 2$.

The co-ordinate of point $= \left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$= \left(\dfrac{3(4) + 2(-1)}{3 + 2}, \dfrac{3(2) + 2(7)}{3 + 2}\right)$

$= \left(\dfrac{12 - 2}{5}, \dfrac{6 + 14}{5}\right)$

$= (2, 4)$

42. Suppose A (-7, 5) and B (5, –1) connecting the line segment AB are the trisection points P and Q.

$\therefore$ AP = PQ = QB

Here, point P divides AB intenally in ration 1 : 2.

$\therefore$ The co-ordinate of point P $= \left(\dfrac{1(5) + 2(-7)}{1 + 2}, \dfrac{1(-1) + 2(5)}{1 + 2}\right)$

$= \left(\dfrac{5 - 14}{3}, \dfrac{-1 + 10}{3}\right)$

$= (-3, 3)$

Same as, point Q divides AB internally in ration 2 : 1.

$\therefore$ The co-ordinate of point Q $= \left(\dfrac{2(5) + 1(-7)}{2 + 1}, \dfrac{2(-1) + 1(5)}{2 + 1}\right)$

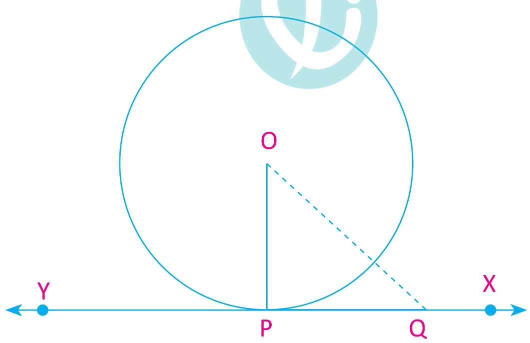
$= \left(\dfrac{10 - 7}{3}, \dfrac{-2 + 5}{3}\right)$

$= (1, 1)$

43. Given : A circle with centre O and a tangent XY to the circle at a point P.

Prove that : OP is perpendicular to XY. i.e. OP $\perp$ XY

Figure :



Proof : Take a point Q on XY other than P and join OQ.

The point Q must lie out side the circle. If Q lies inside the circle, XY will become a secant and not a tangent to the circle.

Therefore, Q is longer than the radius OP of the circle.

i.e., OQ > OP

Since, this happens for every point on line XY except the point P, OP is the shortest of all the distance of the point O to XY.

So, OP $\perp$ XY is perpendicular to XY.

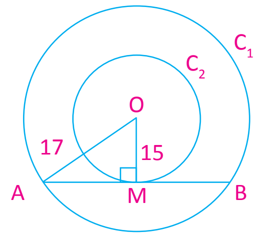
44. The raddi of the two O concentric circles are C_1 and C_2.

Radius of C_1 = OA = r^1 = 17 cm

Radius of C_2 = OM = r^2 = 15 cm

The chord AB of C_1 touches C_2 at the point M.

In ΔOMA, ∠M = 90°



$\therefore$ AM = $\sqrt{OA^2 - OM^2}$

$\therefore$ AM = $\sqrt{(17)^2 - (15)^2}$

$\therefore$ AM = $\sqrt{289 - 225}$

$\therefore$ AM = $\sqrt{64}$

$\therefore$ AM = 8

But, AB = 2 AM

$\therefore$ AB = 2 × 8

$\therefore$ AB = 16

Thus, the length of chord AB is 16 cm.

45.

Number of mangoes (class)	Number of boxes (f_i)	x_i	u_i	$f_i u_i$
50–52	15	51	–2	–30
53–55	110	54	–1	–110
56–58	135	57 = a	0	0
59–61	115	60	1	115
62–64	25	63	2	50
Total	Σf_i = 400	–	–	25 = $\Sigma f_i u_i$

Mean $\bar{x}$ $= a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$\therefore \bar{x} = 57 + \frac{25}{400} \times 3$

$\therefore \bar{x} = 57 + 0.19$

$\bar{x} = 57.19$

So, Mean number of mangoes kept in a packing box is 57.19.

Here, the deviation method is used to find the mean.

46. One shirt is drawn at random from the carton of 100 shirts. Therefore, there are 100 equally likely outcomes.

(i) The number of outcomes favourable (i.e., acceptable) to Jimmy = 88

Therefore,

P (shirt is acceptable to Jimmy) $= \frac{88}{100}$

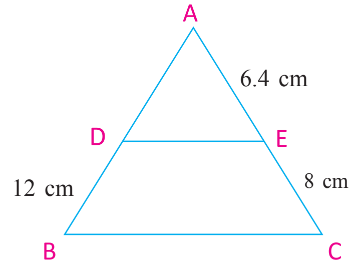
$= 0.88$

(ii) The number of outcomes favourable to Sujatha = 88 + 8 = 96

So, P (shirt is acceptable to Sujatha)

$= \frac{96}{100} = 0.96$

47. $\frac{AD}{DB} = \frac{AE}{EC}$ (Theorem – 6.1)



$\therefore \frac{AD}{12} = \frac{6.4}{8}$

$\therefore AD = \frac{6.4 \times 12}{8}$

$\therefore AD = 9.6$ cm

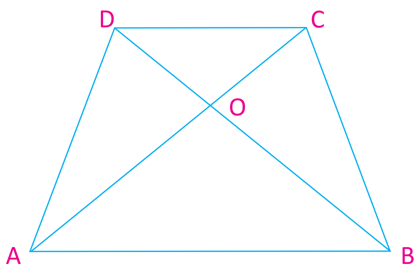
Now, A – D – B,

AB = AD + DB

$\therefore AB = 9.6 + 12$

$\therefore AB = 21.6$ cm

48.



Here, ABCD is a parallelogrum where AB || DC.

$\therefore \angle CAB = \angle ACD$ and $\angle DBA = \angle BDC$ ...(1)

Now, in Δ OAB and Δ OCD,

$\angle OAB = \angle OCD$ and $\angle OBA = \angle ODC$ (As per (1))

$\therefore \Delta OAB \sim \Delta OCD$ (AA criterion)

$\therefore \frac{OA}{OC} = \frac{OB}{OD}$

49. Suppose, the size of the base = x cm

Hence, the measurement of altitude = $(x - 7)$ cm

According to Pythagoras theorem,

$(\text{Base})^2 + (\text{Altitude})^2 = (\text{Hypotenuse})^2$

$\therefore (x)^2 + (x - 7)^2 = (13)^2$

$\therefore x^2 + x^2 - 14x + 49 = 169$

$\therefore 2x^2 - 14x + 49 - 169 = 0$

$\therefore 2x^2 - 14x - 120 = 0$

$\therefore x^2 - 7x - 60 = 0$

$\therefore x^2 - 12x + 5x - 60 = 0$

$\therefore x(x - 12) + 5(x - 12) = 0$

$\therefore (x + 5)(x - 12) = 0$

$\therefore x + 5 = 0$ OR $x - 12 = 0$

$\therefore x = -5$ OR $x = 12$

But the size of the base should not be negative.

The base of the given triangle = 12 cm

The altitude of this triangle will be = 12 – 7 = 5 cm.

50. Here, $n = 50$, $a_{50} = 106$, $a_3 = 12$

$a_{50} = a + 49d = 106$ ...(i)

$a_3 = a + 2d = 12$ ...(ii)

Subtracting equation (ii) from (i), we get,

$(a + 49d) - (a + 2d) = 106 - 12$

$\therefore\ a + 49d - a - 2d = 94$

$\therefore\ 47d = 94$

$\therefore\ d = 2$

Put $d = 2$, in equation (ii) we get,

$a + 2d = 12$

$\therefore\ a + 2(2) = 12$

$\therefore\ a + 4 = 12$

$\therefore\ a = 8$

Now, $a_{29} = a + 28d = 8 + 28(2) = 8 + 56 = 64$

Therefore, the 29th term is 64.

51. Here maximum class frequency is 61 which belongs to class interval 60-80.

$\therefore\ l$ = lower class limit of modal class = 60

$\hbar$ = class size = 20

f_1 = frequency of modal class = 61

f_0 = frequency of class preceding the modal class = 52

f_2 = frequency of class succeeding the modal class = 38

$$\text{Mode } Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times \hbar$$

$$\therefore\ Z = 60 + \left(\frac{61 - 52}{2(61) - 52 - 38}\right) \times 20$$

$$\therefore\ Z = 60 + \frac{9 \times 20}{32}$$

$$\therefore\ Z = 60 + 5.625$$

$$\therefore\ Z = 65.625$$

So, modal life time of electrical componens is 65.625 hours.

52.

(class)	Frequency(f_i)	cf
5 – 14	5	5
14 – 23	11	16
23 – 32	x	$16 + x$
32 – 41	53	$69 + x$
41 – 50	y	$69 + x + y$
50 – 59	16	$85 + x + y$
59 – 68	10	$95 + x + y$

Here, M = 38.2 and $\sum fi = n = 165$

median class = 32 – 41

$\therefore\ l = 32,\ cf = 16 + x,\ f = 53,\ h = 9$

$$M = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\therefore\ 38.2 = 32 + \left(\frac{\frac{165}{2} - (16 + x)}{53}\right) \times 9$$

$$\therefore\ 38.2 - 32 = \left(\frac{82.5 - 16 - x)}{53}\right) \times 9$$

$$\therefore\ 6.2 = \frac{(66.5 - x) \times 9}{53}$$

$$\therefore\ \frac{6.2 \times 53}{9} = 66.5 - x$$

$\therefore\ 36.5 = 66.5 - x$

$\therefore\ x = 66.5 - 36.5$

$\therefore\ x = 30$

Now, $\sum fi = n = 165$

$\therefore\ 95 + x + y = 165$

$\therefore\ 95 + 30 + y = 165$

$\therefore\ 125 + y = 165$

$\therefore\ y = 165 - 125$

$\therefore\ y = 40$

Thus, $x = 30$ and $y = 40$

53. Total numbers of outcomes = 50 + 40 = 90.

(i) Suppose event A is the number of outcomes

chosen for a card with the name of a girl in both class = 20 + 25 = 45

$$\therefore\ P(A) = \frac{45}{90} = \frac{1}{2}$$

(ii) Suppose event B is the number of outcomes

chosen for a card with the name of a boy in both class = 30 + 15 = 45

$$\therefore\ P(B) = \frac{45}{90} = \frac{1}{2}$$

(iii) Suppose event C is the number of outcomes

chosen for a card with the name of a girl of class 10 A = 20

$$\therefore\ P(C) = \frac{20}{90} = \frac{2}{9}$$

(iv) Suppose event D is the number of outcomes

chosen for a card with the name of a boy of class 10 B = 15

$$\therefore\ P(D) = \frac{15}{90} = \frac{1}{6}$$

54. Total numbers of outcomes = 52 – 12 = 40.

(i) Suppose event A the value of the card 7 = 4

$\therefore$ P (A) = $\frac{4}{40} = \frac{1}{10} = 0.1$

(ii) Suppose event A is the value of the card more than 7 = 12 [4 (8), 4 (9), 4 (10)]

$\therefore$ P (B) = $\frac{12}{40} = \frac{3}{10} = 0.3$

(iii) Suppose event C, the value of the card less than 7 = 24 [4 (1), 4 (2), 4 (3), 4 (4), 4 (5), 4 (6)]

$\therefore$ P (C) = $\frac{24}{40} = \frac{6}{10} = 0.6$

(iv) Suppose event D, the value of the card odd figure = 20 [4 (1), 4 (3), 4 (5), 4 (7), 4 (9)]

$\therefore$ P (D) = $\frac{20}{40} = 0.5$